Math 115

Spring 2019

Lecture 26

$? \ a^2 + b^2 = c^2 \ ?$
$y = mx + b \ ? \ d = rt$

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find all excluded Values:

1) $\dfrac{2x^3}{3x+4}$ Deno. = 0, Solve

$3x+4=0$ ⟶ $x = \dfrac{-4}{3}$

$3x=-4$ $\boxed{\text{E.V. } -4/3}$

2) $\dfrac{-8x}{5x-8}$

$5x-8=0$

$5x=8$

$x=8/5$ E.V. $8/5$

3) $\dfrac{2x-5}{x^2-5x-6}$

$x^2-5x-6=0$ ⟶ By Z.F.P.

$(x-6)(x+1)=0$ $x-6=0$ or $x+1=0$

$x=6$ $x=-1$

E.V. $6 \ \&\ -1$

Simplify:

1) $\dfrac{\overset{4}{-\cancel{32}}\ x^{12}}{\underset{5}{\cancel{40}}\ x^3} = \boxed{\dfrac{-4\ x^9}{5}}$

$= \boxed{\dfrac{-4}{5}x^9}$

2) $\dfrac{8z-24}{4z-12} = \dfrac{\overset{2}{\cancel{8}}(\cancel{z-3})}{\underset{1}{\cancel{4}}(\cancel{z-3})}$

$= \dfrac{2}{1} = \boxed{2}$

3) $\dfrac{9x^2-4}{9x+6}$

$= \dfrac{(3x)^2-2^2}{9x+6} = \dfrac{(3x+2)(3x-2)}{3(3\cancel{x+2})}$

$= \boxed{\dfrac{3x-2}{3}}$

$\dfrac{3x}{3} - \dfrac{2}{3} =$

$x - \frac{2}{3}$

4) $\dfrac{x^2-5x-14}{x^2+x-2}$

$= \dfrac{(x-7)(\cancel{x+2})}{(x-1)(\cancel{x+2})}$

$= \boxed{\dfrac{x-7}{x-1}}$

Consider the rational expression $\dfrac{2x^2-3x-5}{2x^2-7x+5}$

1) Find all excluded values

$2x^2-7x+5=0$

$a=2 \quad b=-7 \quad c=5$

$b^2-4ac = (-7)^2-4(2)(5)=9$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-(-7)\pm\sqrt{9}}{2(2)}$

$= \dfrac{7\pm3}{4}$

$x = \dfrac{7+3}{4} \qquad x = \dfrac{7-3}{4}$

$= 5/2 \qquad =1$

E.V. $\dfrac{5}{2}$ & 1

2) Reduce the expression

$\dfrac{2x^2-3x-5}{2x^2-7x+5} = \dfrac{(\cancel{2x-5})(x+1)}{(\cancel{2x-5})(x-1)}$

$= \boxed{\dfrac{x+1}{x-1}}$

Simplify:

1) $\dfrac{5x-15}{3x+9} \cdot \dfrac{4x+12}{6x-18}$

$= \dfrac{5(x-3)}{3(x+3)} \cdot \dfrac{\overset{2}{4}(x+3)}{\underset{3}{6}(x-3)}$

$= \boxed{\dfrac{10}{9}}$

2) $\dfrac{x^2-4}{8x-16} \div \dfrac{x+2}{8}$

$= \dfrac{(x+2)(x-2)}{8(x-2)} \cdot \dfrac{8}{x+2}$

$= \dfrac{x-2}{x-2} \qquad = \boxed{1}$

Simplify:

1) $\dfrac{2x^2+7x+3}{x^2-9} \cdot \dfrac{x^2-3x}{2x^2+11x+5}$

$= \dfrac{(2x+1)(x+3)}{(x+3)(x-3)} \cdot \dfrac{x(x-3)}{(2x+1)(x+5)}$

$= \boxed{\dfrac{x}{x+5}}$

2) $\dfrac{2x^2-5x-12}{x^2-10x+24} \div \dfrac{4x^2-9}{x^2-9x+18}$

$= \dfrac{2x^2-5x-12}{x^2-10x+24} \cdot \dfrac{x^2-9x+18}{4x^2-9}$

$= \dfrac{(2x+3)(x-4)}{(x-4)(x-6)} \cdot \dfrac{(x-6)(x-3)}{(2x+3)(2x-3)}$

$= \boxed{\dfrac{x-3}{2x-3}}$

Simplify

1) $\dfrac{x+2}{x+3} + \dfrac{1}{x+3}$

$= \dfrac{x+2+1}{x+3} = \dfrac{x+3}{x+3} = \boxed{1}$

2) $\dfrac{2x-5}{x-4} - \dfrac{x-1}{x-4}$

$= \dfrac{2x-5-x+1}{x-4} = \dfrac{x-4}{x-4} = \boxed{1}$

3) $\dfrac{x^3}{x^2-9} - \dfrac{3x^2}{x^2-9}$

$= \dfrac{x^3-3x^2}{x^2-9} = \dfrac{x^2(x-3)}{(x-3)(x+3)}$

$= \boxed{\dfrac{x^2}{x+3}}$

4) $\dfrac{x^2}{x^2-4} + \dfrac{2x}{x^2-4}$

$= \dfrac{x^2+2x}{x^2-4}$

$= \dfrac{x(x+2)}{(x-2)(x+2)} = \boxed{\dfrac{x}{x-2}}$

5) $\dfrac{5}{x-2} - \dfrac{3}{x+4} = \dfrac{5(x+4)}{(x-2)(x+4)} - \dfrac{3(x-2)}{(x+4)(x-2)}$

$= \dfrac{5(x+4)-3(x-2)}{(x-2)(x+4)}$

$= \dfrac{5x+20-3x+6}{(x-2)(x+4)}$

$= \dfrac{2x+26}{(x-2)(x+4)} = \boxed{\dfrac{2(x+13)}{(x-2)(x+4)}}$

6) $\dfrac{x}{x-8} - \dfrac{2}{x+6}$

$= \dfrac{x(x+6)}{(x-8)(x+6)} - \dfrac{2(x-8)}{(x+6)(x-8)}$

$= \dfrac{x(x+6) - 2(x-8)}{(x-8)(x+6)} = \dfrac{x^2 + 6x - 2x + 16}{(x-8)(x+6)}$

$= \boxed{\dfrac{x^2 + 4x + 16}{(x-8)(x+6)}}$

Simplify

$\dfrac{3}{x^2 + 3x - 4} - \dfrac{2}{x^2 + 7x + 12}$

$= \dfrac{3(x+3)}{(x+4)(x-1)(x+3)} - \dfrac{2(x-1)}{(x+4)(x+3)(x-1)}$

$= \dfrac{3(x+3) - 2(x-1)}{(x+4)(x-1)(x+3)} = \dfrac{3x + 9 - 2x + 2}{(x+4)(x-1)(x+3)}$

$= \dfrac{x + 11}{(x+4)(x-1)(x+3)}$

Simplify: $\dfrac{6}{x^2+3x} - \dfrac{1}{x^2-x} + \dfrac{2}{x^2+2x-3}$

$= \dfrac{6(x-1)}{x(x+3)(x-1)} - \dfrac{1(x+3)}{x(x-1)(x+3)} + \dfrac{2x}{(x+3)(x-1)x}$

$= \dfrac{6(x-1) - 1(x+3) + 2x}{x(x+3)(x-1)} = \dfrac{6x-6-x-3+2x}{x(x+3)(x-1)}$

$= \boxed{\dfrac{7x-9}{x(x+3)(x-1)}}$

Complex Fraction: It is a fraction that contains at least one fraction

$\dfrac{2-\frac{1}{4}}{\frac{1}{2}}$ $\qquad$ $\dfrac{4+\frac{1}{x}}{2-\frac{1}{x}}$ $\quad,\quad$ $\dfrac{\frac{x}{4}-\frac{4}{x}}{\frac{x}{2}+\frac{1}{2}}$

To simplify a Complex fraction

1) Find LCD of all denominators.

2) Multiply everything by the LCD.

3) Simplify, simplify, simplify.

Simplify $\dfrac{3 - \frac{1}{4}}{1 + \frac{1}{2}} = \dfrac{4 \cdot 3 - 4 \cdot \frac{1}{4}}{4 \cdot 1 + \overset{2}{4} \cdot \frac{1}{2}}$

LCD = 4

$= \dfrac{12 - 1}{4 + 2} = \boxed{\dfrac{11}{6}}$

Simplify:

$\dfrac{\frac{2}{5} - \frac{1}{3}}{\frac{1}{5} + \frac{2}{3}} = \dfrac{\overset{3}{15} \cdot \frac{2}{5} - \overset{5}{15} \cdot \frac{1}{3}}{\overset{3}{15} \cdot \frac{1}{5} + \overset{5}{15} \cdot \frac{2}{3}} = \dfrac{6 - 5}{3 + 10} = \boxed{\dfrac{1}{13}}$

LCD = 15

Simplify:

$\dfrac{x - \frac{4}{x}}{1 + \frac{2}{x}} = \dfrac{x \cdot x - x \cdot \frac{4}{x}}{x \cdot 1 + x \cdot \frac{2}{x}}$

LCD = x

$= \dfrac{x^2 - 4}{x + 2}$

$= \dfrac{(x+2)(x-2)}{x+2}$

$= \boxed{x - 2}$

Simplify:

$$\frac{\frac{2}{x-1} + 2}{\frac{2}{x-1} - 2} = \frac{(x-1) \cdot \frac{2}{x-1} + (x-1) \cdot 2}{(x-1) \cdot \frac{2}{x-1} - (x-1) \cdot 2}$$

$$LCD = x-1 \qquad = \frac{2 + 2(x-1)}{2 - 2(x-1)}$$

$$= \frac{2 + 2x - 2}{2 - 2x + 2} = \frac{2x}{4 - 2x} = \frac{2x}{2(2-x)}$$

$$= \boxed{\frac{x}{2-x}}$$

Simplify

$$\frac{1 + \frac{4}{x} + \frac{3}{x^2}}{1 - \frac{9}{x^2}} = \frac{x^2 + 4x + 3}{x^2 - 9}$$

$$LCD = x^2 \qquad = \frac{(x+3)(x+1)}{(x+3)(x-3)} = \boxed{\frac{x+1}{x-3}}$$

Simplify

$$\frac{\dfrac{3}{x-3} - \dfrac{2}{x+2}}{\dfrac{1}{x^2-x-6}} = \frac{\dfrac{3}{x-3} - \dfrac{2}{x+2}}{\dfrac{1}{(x-3)(x+2)}}$$

LCD = $(x-3)(x+2)$

$$= \frac{3(x+2) - 2(x-3)}{1} = \frac{3x+6-2x+6}{1} = \boxed{x+12}$$

Simplify $\quad \dfrac{4 - 9x^{-2}}{2 + 3x^{-1}} \qquad$ Recall

$$x^{-n} = \frac{1}{x^n}$$

$$= \frac{4 - \dfrac{9}{x^2}}{2 + \dfrac{3}{x}} = \frac{4x^2 - 9}{2x^2 + 3x} = \frac{(2x+3)(2x-3)}{x(2x+3)}$$

LCD = x^2

$$= \boxed{\frac{2x-3}{x}}$$

$$\frac{2x}{x} - \frac{3}{x} = \boxed{2 - \frac{3}{x}} \checkmark \qquad\qquad \neq \frac{2-3}{x} \neq \frac{-1}{x}$$

Simplify:

Recall $x^{-n} = \dfrac{1}{x^n}$

$$\frac{6 + 11x^{-1} - 10x^{-2}}{3 - 14x^{-1} + 8x^{-2}} = \frac{6 + \dfrac{11}{x} - \dfrac{10}{x^2}}{3 - \dfrac{14}{x} + \dfrac{8}{x^2}}$$

LCD $= x^2$

$$= \frac{6x^2 + 11x - 10}{3x^2 - 14x + 8} = \frac{(3x-2)(2x+5)}{(3x-2)(x-4)}$$

$$= \boxed{\frac{2x+5}{x-4}}$$

$6x^2 + 11x - 10$

$P = -60$
$S = 11$
$15, -4$

-60

$\underbrace{6x^2 + 15x}_{3x(2x+5)} \underbrace{-4x - 10}_{-2(2x+5)} = (2x+5)(3x-2)$

Ch. 2

Solve: $\dfrac{x}{2} - \dfrac{1}{3} = \dfrac{5}{6}$

$6 \cdot \dfrac{x}{2} - 6 \cdot \dfrac{1}{3} = 6 \cdot \dfrac{5}{6}$

LCD $= 6$

$3x - 2 = 5$

$3x = 7$

$\boxed{x = \dfrac{7}{3}} \Rightarrow \left\{ \dfrac{7}{3} \right\}$

Solve $2 - \dfrac{4}{x} = 5$

$2x - 4 = 5x$

LCD $= x$

$2x - 5x = 4$

$-3x = 4$

$x = -\dfrac{4}{3}$

$\Rightarrow \left\{ \dfrac{-4}{3} \right\}$

Solve $\quad 1 + \dfrac{1}{x} = \dfrac{30}{x^2}$.

$\qquad LCD = x^2$

$\qquad x^2 + x = 30$

$\qquad x^2 + x - 30 = 0$

$\qquad (x+6)(x-5) = 0$

$\qquad\quad$ by Z.F.P. $\qquad x+6=0 \qquad x-5=0$

$\qquad\qquad\qquad\qquad\qquad\quad x=-6 \qquad\quad x=5$

$$\{-6, 5\}$$

Solve! $\quad 6 = \dfrac{7}{x} + \dfrac{3}{x^2}$

$LCD = x^2$

$6x^2 = 7x + 3 \qquad\qquad 6x^2 - 7x - 3 = 0$

$\qquad\qquad\qquad\qquad\qquad a=6 \quad b=-7 \quad c=-3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad b^2-4ac = (-7)^2 - 4(6)(-3)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 49 + 72$

$x = \dfrac{-(-7) \pm \sqrt{121}}{2(6)} \qquad\qquad\qquad = 121$

$\qquad\qquad\qquad\qquad x = \dfrac{7+11}{12} \qquad x = \dfrac{7-11}{12} \qquad \left\{\dfrac{-1}{3}, \dfrac{3}{2}\right\}$

$\quad = \dfrac{7 \pm 11}{12} \qquad\qquad = \dfrac{18}{12} \qquad\qquad = \dfrac{-4}{12} = \boxed{\dfrac{-1}{3}}$

$\qquad\qquad\qquad\qquad\qquad\qquad = \boxed{3/2}$

Solve $\dfrac{3}{x+3} - \dfrac{2}{x-3} = \dfrac{-15}{x^2-9}$

$LCD = (x+3)(x-3)$

$3(x-3) - 2(x+3) = -15$

$3x - 9 - 2x - 6 = -15$

$x - 15 = -15 \qquad \longrightarrow \{0\}$

$\boxed{x = 0}$

The area of a rectangular shape is $70\,ft^2$.
The length is 1 ft shorter than 3 times
its width.
Find its dimensions.

$A = 70$
$LW = 70$

$x(3x-1) = 70$
$3x^2 - x - 70 = 0$
$a = 3 \quad b = -1 \quad C = -70$

$b^2 - 4ac = (-1)^2 - 4(3)(-70)$
$= 1 + 840 = 841$

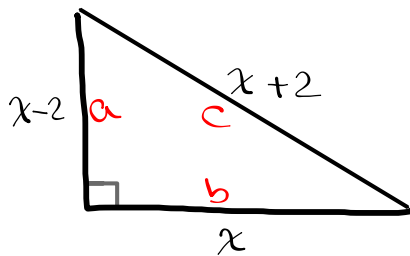$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-1)\pm\sqrt{841}}{2(3)} = \dfrac{1 \pm 29}{6}$

$x = \dfrac{1 + 29}{6} \qquad x = \dfrac{1-29}{6} = \cancel{\dfrac{-28}{6}}$

$= 30/6 \qquad x = 5$

$3x - 1$

x

5ft, 14ft

find all three sides:

Right Triangle
Pythagorean Thrm

$$a^2 + b^2 = c^2$$

$$(x-2)^2 + x^2 = (x+2)^2$$

$$(x-2)(x-2) + x^2 = (x+2)(x+2)$$

$$x^2 - 2x - 2x \cancel{+4} + x^2 = x^2 + 2x + 2x \cancel{+4}$$

$$\underbrace{}_{4x}$$

$$x^2 - 4x - 4x = 0$$

$$x^2 - 8x = 0$$

$x(x-8) = 0$

$\cancel{x=0}$ $\quad x = 8$

SG 18 → Thursday